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Fluid dynamic force model for rotors with seals or lightly-loaded bearings

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Notation

D, K_B	Fluid film radial viscous damping and stiffness, respectively
D_S, K, M	Rotor modal external damping, stiffness, and mass, respectively
$F_\zeta = F_\xi + jF_\eta$	Fluid dynamic force in coordinates rotating at the rate $\lambda\Omega$.
j	$\sqrt{-1}$
t	Time
$z(t) = x(t) + jy(t)$	Rotor lateral displacement with its horizontal (x) and vertical (y) components in stationary coordinates.
$\zeta(t) = \xi(t) + j\eta(t)$	Rotor lateral displacement with its orthogonal components ξ, η in the coordinates rotating at the rate $\lambda\Omega$
λ	Fluid circumferential average velocity ratio
Ω	Rotor rotative speed
$\bullet$	$\frac{d}{dt}$
$\bullet\bullet$	$\frac{d^2}{dt^2}$

This article summarizes the results of experimental modal testing of rotors in fluid-lubricated bearings and seals which was performed at Bently Rotor Dynamics Research Corporation (BRDRC) during the last ten years [Ref. 1]. A new and efficient model of the fluid forces in seals and lightly-loaded, fully lubricated bearings emerged from these results [Ref. 2, 3]. Used together with multi-mode, modal models of rotors, this new rotor/fluid system model opens new paths for the prediction of rotor stable rotation and the prevention of damaging self-excited lateral fluid whirl and whip vibrations [Ref. 4, 5].

Fluid force model

The physical situation of the rotor/bearing or rotor/seal system can be modeled by two coaxial cylinders, one softly supported and rotating inside the other, stationary cylinder. The relatively small clearance between these cylinders is filled with fluid (gases included). Due to the rotation of the inner cylinder[1], and, due to friction, the fluid within the clearance becomes involved in circumferential motion.

Fluid dynamic forces in the clearance can be modeled in the form of a dashpot and a spring, as is illustrated in Figure 1 (for the moment, we will ignore the fluid inertia). The fluid film-related dashpot and spring are not, however, stationary, but rotate at the rate $\lambda\Omega$, a fraction of the rotative speed Ω (capital omega). The fraction parameter, λ (lambda), has a physical meaning; it is the fluid circumferential average velocity ratio. Figure 2 illustrates the fluid velocity profile and the average circumferential angular velocity.

Introduce two systems of reference coordinates: X and Y are stationary coordinates, and the coordinates ξ (xi) and η (eta) are coordinates that rotate together with the dashpot and spring system, at the fluid average circumferential angular velocity $\lambda\Omega$. The fluid force components can be simply expressed in these rotating coordinates as:

$$F_\xi = D\dot{\xi} + K_B\xi \qquad \textbf{(1)}$$
$$F_\eta = D\dot{\eta} + K_B\eta$$

or

$$F_\zeta = F_\xi + jF_\eta = D\dot{\zeta} + K_B\zeta, \quad j = \sqrt{-1}, \ \zeta(t) = \xi(t) + j\eta(t) \qquad \textbf{(2)}$$

where t is time; F_ζ is the fluid force in the rotating coordinates $\xi\eta$; D is fluid viscous damping; K_B is fluid film radial ►

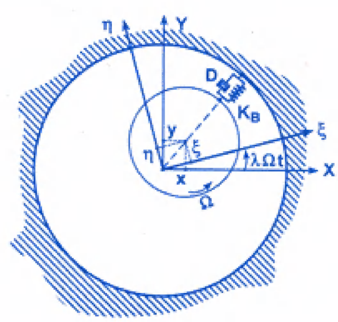

Figure 1
Fluid force generated inside the clearance: the spring and dashpot rotate at the rotative speed $\lambda\Omega$.

1 A very similar situation occurs when the outer cylinder rotates, and the inner one is stationary, or when both cylinders rotate at different speeds, such as in the case of partially engaged hydraulic clutches.

stiffness; ζ(zeta) is the rotor lateral displacement using complex rotating coordinates and $\cdot$ is $\frac{d}{dt}$.

Traditionally, the dashpot damping force is proportional to the velocity of the dashpot end at the rotor surface, and the stiffness force is proportional to the spring end displacement. Note, however, that the spring and dashpot ends are not attached, either to the bearing or to the rotor surfaces, but slide with the velocity $\lambda\Omega(R+c)$ on the bearing and $(1-\lambda)\ \Omega R$ on the rotor, where R is the rotor radius and c is the radial clearance.

Rotor model

Consider the rotor itself (Figure 3). The isotropic, balanced rotor is represented by its first lateral mode modal parameters: stiffness K, mass M, and external damping, D_S. Omitting the fluid force for a moment, the rotor model — its equation of motion, a balance of forces expressed in the stationary coordinates XY, is:

$$M\ddot{x}+D_S\dot{x}+Kx=0$$
$$M\ddot{y}+D_S\dot{y}+Ky=0$$

or

$$M\ddot{z}+D_S\dot{z}+Kz=0 \qquad z=x+jy \tag{3}$$

where $z(t)=x(t)+jy(t)$ is the rotor lateral complex deflection and $\cdot\cdot$ is $\frac{d^2}{dt^2}$.

Eq. (3) can be transformed into rotating coordinates $\xi\eta$, using the following transformation algorithm:

$$x=\xi\cos\lambda\Omega t-\eta\sin\lambda\Omega t$$
$$y=\xi\sin\lambda\Omega t+\xi\cos\lambda\Omega t$$

or, further applying Euler's identity $e^{j\alpha}\equiv\cos\alpha+j\sin\alpha$:

$$z=\zeta e^{j\lambda\Omega t};\quad z=x+jy,\quad \zeta=\xi+j\eta \tag{4}$$

For Eq. (3) transformation, the velocity $\dot{z}$ and acceleration $\ddot{z}$ are required:

$$\dot{z}=\frac{dz}{dt}=(\dot{\zeta}+j\lambda\Omega\zeta)e^{j\lambda\Omega t}$$

$$\ddot{z}=\frac{d^2z}{dt^2}=(\ddot{\zeta}+2j\lambda\Omega\dot{\zeta}-\lambda^2\Omega^2\zeta)e^{j\lambda\Omega t}$$

Eq. (3) in the coordinates rotating at the rate $\lambda\Omega$ will have, therefore, the following form:

$$M(\ddot{\zeta}+2j\lambda\Omega\dot{\zeta}-\lambda^2\Omega^2\zeta)+D_s(\dot{\zeta}+j\lambda\Omega\zeta)+K\zeta=0 \tag{5}$$

The first term in Eq. (5) is composed of three parts; the relative inertia force, the Coriolis inertia force, and the centripetal inertia force.

Model of a rotor with fluid interaction

Now Eq. (5) can be supplemented by the fluid force (2), as all forces are expressed now in the same coordinate system. In rotating coordinates $\xi\eta$, the rotor/fluid model will have the following form:

$$M(\ddot{\zeta}+2j\lambda\Omega\dot{\zeta}-\lambda^2\Omega^2\zeta)+D_s(\dot{\zeta}+j\lambda\Omega\zeta)+K\zeta+D\dot{\zeta}+K_B\zeta=0 \tag{6}$$

To transform the rotor/fluid model back to the stationary coordinates, the inverse transformation (4) must be used:

$$\zeta=ze^{-j\lambda\Omega t} \tag{7}$$

The rotor/fluid model in the stationary coordinates has its final form:

$$M\ddot{z}+D_S\dot{z}+Kz+D(\dot{z}-j\lambda\Omega z)+K_Bz=0 \tag{8}$$

The last three terms of Eq. (8) represent the rotating fluid force.

When the lateral complex coordinate z is split back to the horizontal x and vertical y displacements, Eq. (8) will have the following familiar form:

$$\begin{aligned}M\ddot{x}+D_S\dot{x}+Kx+D(\dot{x}+\lambda\Omega y)+K_Bx=0\\ M\ddot{y}+D_S\dot{y}+Ky+D(\dot{y}-\lambda\Omega x)+K_By=0\end{aligned} \tag{9}$$

or, using the matrix notation:

$$\begin{bmatrix}M & 0\\ 0 & M\end{bmatrix}\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix}+\begin{bmatrix}D_S+D & 0\\ 0 & D_S+D\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}+\begin{bmatrix}K+K_B & \lambda\Omega D\\ -\lambda\Omega D & K+K_B\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=0 \tag{10}$$

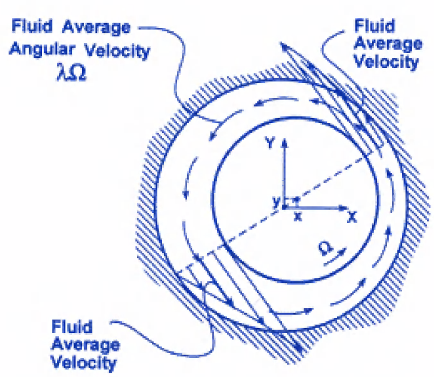

Figure 2
Fluid velocity profile inside the bearing or seal clearance.

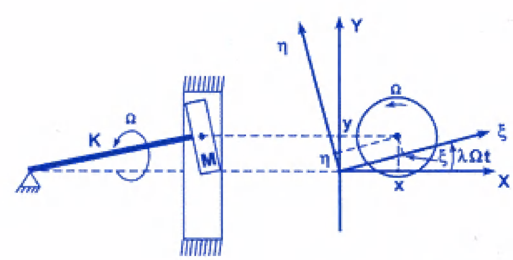

Figure 3
Rotor model (bearing clearance exaggerated).

The last matrix in Eq. (10) is known as the "stiffness matrix," since it stands as a multiplier of the rotor displacement vector. The diagonal terms of this matrix are, indeed, the stiffnesses of the rotor and of corresponding fluid film in the x and y directions. The off-diagonal terms in this matrix are known as "cross-coupled stiffnesses," and are responsible for rotor instability. As can be seen, these skew-symmetric terms are products of fluid damping D, λ, and rotative speed Ω, each of them having a clear physical sense. The "cross-coupled stiffness" is simply a result of fluid viscous damping rotation at the circumferential velocity $\lambda\Omega$.

Fluid-force model with fluid inertia effect

Consider now that the fluid force (2) contains, not only the spring and dashpot, but also an inertia element (Fig. 4):

$$F_{\zeta} = M_f \ddot{\zeta} + D\dot{\zeta} + K_B \zeta \qquad \textbf{(11)}$$

where M_f is the fluid inertia effect, and F_{ζ} is the fluid force rotating at the rate $\lambda\Omega$.

Following the same reasoning used previously, the rotor/fluid system model in the stationary coordinates is as follows:

$$M\ddot{z} + D_S \dot{z} + Kz + M_f(\ddot{z} - 2j\lambda\Omega\dot{z} - \lambda^2\Omega^2 z) + D(\dot{z} - j\lambda\Omega z) + K_B z = 0 \qquad \textbf{(12)}$$

or in the matrix form:

$$\begin{bmatrix} M+M_f & 0 \\ 0 & M+M_f \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} D_s+D & 2M_f\lambda\Omega \\ -2M_f\lambda\Omega & D_S+D \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} K+K_B-M_f\lambda^2\Omega^2 & \lambda\Omega D \\ -\lambda\Omega D & K+K_B-M_f\lambda^2\Omega^2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \qquad \textbf{(13)}$$

As it can be seen, some new fluid-inertia related terms appeared in the rotor/fluid system model equations (12) and (13). The damping matrix in Eq. (13) is now complemented by the skew-symmetric, "gyroscopic"-like terms due to the Coriolis acceleration of the rotating fluid. The stiffness matrix diagonal terms are decreased by fluid inertia centrifugal force, since $M_f\lambda^2\Omega^2$ appears with the minus sign. This implies that the resulting radial "stiffness" may become negative at some high rotative speed Ω. And it really does, which was shown during modal perturbation testing [6].

The question remains whether the fluid damping and fluid inertia rotate at the same rate $\lambda\Omega$. Most probably, in a general case, these rates are different [7], that is, Eq. (12) should be rewritten as follows:

$$M\ddot{z} + D_S \dot{z} + Kz + M_f(\ddot{z} - 2j\lambda_f\Omega^2\dot{z} - \lambda_f^2\Omega^2 z) + D(\dot{z} - j\lambda\Omega z) + K_B z = 0 \qquad \textbf{(14)}$$

where $\lambda_f\Omega$ is the angular velocity at which fluid inertia force rotates. The results of several experiments indicate that λ_f is up to 10% larger than λ, especially at higher rotative speeds [7].

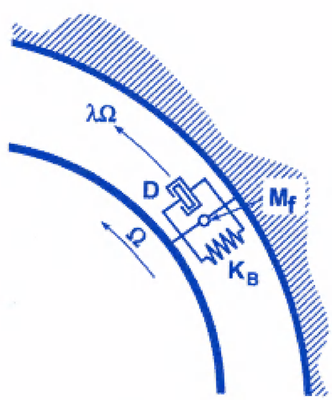

Figure 4
Fluid force model with fluid inertia effect.

Final remarks

The model of a rotor with fluid interaction (14) is still the simplest, as it assumes only one isotropic lateral mode of the rotor, and the fluid force action at the rotor modal mass location. The model can be extended to include more lateral modes of the rotor, and to consider rotor and fluid force anisotropy. Finally, the fluid force, which was assumed here to be linear with displacement and velocity, should be expanded to include its nonlinearity, especially the nonlinearity versus displacement.

This article shows how to start building the rotor/fluid models. It emphasizes the physical meaning of involved parameters, and treats the rotor and fluid as one system. The model in the form of a differential equation is only the first step. It has to be solved in order to trace the actual behavior of the rotor, its lateral displacement under the action of the fluid force. The solution of Eq. (14) will be discussed in a following article.

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